3. A Comparison of the Mathematics Curriculum Documents of Major Asian and Western Countries with Those in Hong Kong

In 1997 and 1998, the Hong Kong Curriculum Development Council (CDC) released the mathematics syllabuses (draft) for primary and secondary schools respectively. These were recommended for use in schools starting 2001. In this section, the aims of the mathematics curriculum, scope and treatment of content, and modes of courses and the curriculum development process in a selected group of countries will be compared with the case in Hong Kong based on an analysis of the relevant curriculum documents.

3.1 The Aims of the Mathematics Curriculum

Hoyles et al (1999) pointed out that there are two complementary categories of aims: those furthering the development of society and those furthering the development of the individual. Wong & Wong (1997), through a detailed analysis of the mathematics curricula in 10 countries and regions, summarized that the educational goal of mathematics curricula in these regions. These include (1) practical; (2) disciplinary; and (3) cultural aims.

Practical aims include:

- Developing the ability to apply mathematics to daily life situations(mathematics literacy);
- Equipping students with the mathematics needed in a majority of professions; and
- Providing a mathematical foundation for further study in science and other related disciplines.

Disciplinary aims include:

- Enabling students to manipulate numbers, symbols and other mathematical objects;
- Developing number sense, symbol sense, spatial sense and a sense of measurement as well as the relation of structure and pattern;
- Facilitating reasoning, deduction and logical thinking;
- facilitating the ability to utilize mathematics to conceptualize, formulate and solve problems, and
- Enabling students to express ideas using the language of mathematics.

Culture aims include:

- Developing an appreciation of the aesthetic nature of mathematics, and
- Developing an awareness of the role of mathematics in various cultures form ancient to modern times, and its relation to other disciplines.

Basing on the above and with reference to the following documents, here we compare the aims of mathematics education in another set of eight countries or regions.

United States :

NCTM (1989). *Curriculum and Evaluation Standards for School Mathematics*.
NCTM (1991). *Professional Standards for Teaching Mathematics*.
NCTM (1995). *Assessment Standards for School Mathematics*.
Supplemented by TIMSS results and
NCTM (1998). Standard 2000: *Principles and Standards for School Mathematics*(draft).

United Kingdom:

Cockcroft, W.H.(Chairman)(1982).Mathematics Counts, London: HMSO
Department of Education and Science and the Welsh office(1991). Mathematics in the National Curriculum, London: HMSO
Department of Education and Science and the Welsh office(1995). Mathematics in the National Curriculum, London: HMSO

Australia:

Australian Education Council(1990). A National Statement on Mathematics for Australian Schools, Carlton, Vic.: Curriculum Corporation.
Australian Education Council(1994a).Mathematics-A Curriculum Profile for Australian School, Carlton, Vic.: Curriculum Corporation.
Australian Education Council(1994b). Mathematics-Work Samples, Carlton, Vic.: Curriculum Corporation.

Singapore:

Curriculum Planning Division(1990a).Mathematics Syllabus: Primary, Singapore: Author.
Curriculum Planning Division(1995).Mathematics Syllabus: Secondary, Singapore: Author.
Curriculum Planning Division(1994).Mathematics Syllabus: Primary 5 and 6(EM3 Stream), Singapore: Author.
Curriculum Planning &Development Division(1998).Mathematics Syllabus: For Implement from January 1999, Singapore: Author.

Korea:

Ministry of Education(1992). The School Curriculum of the Republic of Korea, Seoul: Ministry of Education.
Ministry of Education(1998). The School Curriculum of the Republic of Korea, Seoul: Ministry of Education.

Mainland China:

《九年義務教育 全日制 小學數學教學大綱》（中華人民共和國教育委員會,1992）.
《九年義務教育 全日制 初級中學數學教學大綱》(中華人民共和國教育委員會,1992）.
《 九年義務教育 全日制 高級中學數學教學大綱》（中華人民共和國教育委員會,1996)
《進入二十一世紀中小學數學教育行動綱領 》(進入二十一世紀中小學數學教育行動綱領研究小組,1997）.
《全日制九年制義務教育課程標准》(上海市課程改革委員會，1991)

Taiwan:

《高級中學課程標準》正中書局印行．中華民國教育部國民教育司（1983）
《國民小學課程標準》教育部編印． 中華民國教育部國民教育司（1993）
《國民中學課程標準》教育部編印． 中華民國教育部國民教育司（1994）

Hong Kong

The Curriculum Development Committee (1983), Syllabuses for Primary Secondary Schools Mathematics

The Curriculum Development Committee (1985), Syllabuses for Secondary Schools, Mathematics(Form I to V)

The Curriculum Development Council(1995a), TOC Programme of Study for Schools Mathematics for Key Stage 1

The Curriculum Development Council(1995b), TOC Programme of Study for Schools Mathematics for Key Stage 2

The Curriculum Development Council(1997), (Draft) Framework for Primary Schools Mathematics

The Curriculum Development Council(1998), (Draft) Framework for Secondary Schools Mathematics(S1-S5)

A comparison of the aims of mathematics education in the eight countries or regions is shown in Tables 1(a) and 1(b), and a comparison of the dimensions of mathematics curriculum contents is shown in table 2.

Table 1(a). Aims of Mathematics Education in Eight Countries or Regions

Country	Aims
Hong Kong	**<u>Primary School</u>**: 1.to stimulate the interest of children in the learning of mathematics, to foster a good learning habit and the spirit in thinking independently and overcoming difficult; 2.to develop children's ability in logical thinking, and formulating and solving problems; 3.to induce children to understand and grasp the basic mathematical knowledge and computation skills, and to cultivate the number sense and spatial sense; 4.to encourage children to apply mathematics to solve daily problem; 5.to develop children's ability in using mathematics language as a communication tools; 6.to encourage an appreciation of the pattern and structure of numbers and shapes, and to foster creativity in children. **<u>Secondary School</u>**: 1.the ability to conceptualize, inquire, reason and communicate mathematically, and to use mathematics to formulate and solve problems in daily life as well as in mathematical contexts; 2.the ability to manipulate number, symbols and other mathematical objects; 3.the number sense, symbol sense and a sense of measurement as well as the appreciation of structure and pattern; 4.a positive attitudes towards mathematics and the appreciation of the aesthetic and culture aspect of mathematics.
Mainland China	<u>小學</u>: 使學生理解、掌握數學關系和几何圖形的最基礎的知識；使學生具有進行整數、小數、分數四則計算的能力，培養 初步的邏輯思維能力和空間觀念，能夠運用所學的知識解決簡單的實際問題；使學生受到思想品德教育 . <u>中學</u>: 使學生學好當代社會中每一個公民适應日常生活、參加生產和進一步學習所必需的代數、几何的基礎知識与基本技能，進一步培養運算能力，發展邏輯思維能力和空間觀念，并能夠運用所學的知識解決簡單的實際問題培養學生良好的個性品質和初步的辯証唯物主義的觀點 .
Taiwan	<u>小學</u>：養成主動地從自己的經驗中，建构与理解數學的概念，并透過了解及評价別人解題方式的過程，進 而養成尊重別人觀點的態度；養成從數學的觀點考慮周遭事物，并運用數學知識与方法解決問題的能力；培養以數學語言溝通、討論、講道理和批判事物的精神；養成在日常生活中善用各類工具從事學習及解決問題的習慣 . <u>中學</u>：引導學生認識數學在生活中的功用，以提高學習的興趣；輔導學生獲得數、量、形的基本知識与技能，以提升數學素養；培養學生通用數學方法解決問題的習慣与能力；啟發學生思考、推理与創造的能力，培養學生主動學習的態度及欣賞數學的能力 .
Korea	1.through the mathematical experiences in investigation various phenomena faced in evade life, students will be able to understand basic mathematical concepts, principles and rules, and also the relationship among them; 2.by practicing and applying basic mathematical knowledge and skills to every day problems, students will be able to observe, analyze, organize, and consider the problems mathematically for the purpose of the solution; 3.students will be able to continuously keep interest and concern over mathematics. Also, they will acquire positive attitudes, which may then lead them toward rational dealing with various problem by applying mathematical knowledge and skill that they have already obtained

Table 1(b). Mathematics curriculum goal among eight countries and regions (continued)

Countries	Aims
USA	**<u>NCTM' S standards(</u>1998):** **<u>The Equity Principle</u>**: mathematics instructional programs should promote the learning of mathematics by all students **<u>Mathematics Curriculum Principle</u>**: Mathematics instructional programs should emphasize important and meaningful mathematics through curricula that are coherent and comprehensive. **<u>Teaching Principle</u>**: Mathematics instructional programs depend on competent and caring teachers who teach all students to understand and use mathematics. **<u>Learning Principle</u>**: Mathematics instructional programs should enable all students to understand and use mathematics. **<u>Assessment Principle</u>**: Mathematics instructional programs should include assessment to monitor, enhance, and evaluate the mathematics learning of all students and to inform teaching. **<u>Technology Principle</u>**: Mathematics instruction al programs should use technology to help all students understand mathematics and should prepare them the use mathematics in an increasingly technological world. **<u>NCTM' S Standards</u>**(1989): 1. tudents learn to value mathematics; 2. students become confident in their ability to do mathematics; 3. student become mathematics problems solvers; 4. students learn to communicate mathematically, and 5. students learn to reason mathematical.
UK[1]	1 develop positive attitudes to the learning and application of mathematics 2. develop ability and confidence in the application of mathematics 3. develop an appreciation of the nature of mathematics and mathematical process, of how mathematical ideas help in interpreting the world in which they live; of aesthetic and historical aspects of mathematics 4. develop their ability to communicate mathematical ideas both orally and in writing; read and comprehend a piece of mathematics; 5. acquire a sound base of the knowledge skills and attitudes required for further study in mathematics, in other subjects and in employment; 6. develop skill of modeling, generalization and interpretation of results relevant to both application and development of mathematics. 7. develop learning and thinking skills of more general application, for example in decision making; 8. develop an ability in the appropriate mathematical use of calculators and microcomputers, including the use of various software packages; 9. develop an ability to argue logically and understand the nature of rigor; 10.acquire strategies for the solution of extended problems in mathematics.
Australia	1. students should develop confidence and competence in dealing with commonly occurring situation; 2. students should develop positive attitudes towards their involvement in mathematics; 3. students should develop their capacity to use mathematics in solving problems individually and collaboratively; 4. students should learn to communicate mathematically; 5 students should learn techniques and tools which reflect modern mathematics; 6. students should learn experience the processes through which mathematics develops.
Singapore	1. acquire the necessary mathematical knowledge and skill, develop thinking processes and apply them in mathematical situations that they will meet in life; 2. use mathematics as a means of communication; 3. develop positive attitudes and a sense of personal achievement in mathematics; 4. appreciate the importance and power of mathematics in the world around them

[1] Hoyles, C., Morgan, C., & Woodhouse, G. (1999). *Rethinking the Mathematics Curriculum.* London: Falmer Press.p83-84. These are parts of aims for SMP 16-19 exam, and reflect partly the aims of mathematics education, because there are not clear statement about aims of mathematics curriculum in England.

Table 2. Dimensions of Mathematics Curriculum contents

Country	Contents	Processes
USA	Number and Operation; Patterns, Functions, and Algebra; Geometry and Spatial Sense; Measurement; Data Analysis, Statistics, and Probability	Problem solving; Reasoning and proof; Communication; Connections; Representation
UK	Number; Shape, Space and Measures; handling Data; (stage 1~4) Algebra (stage 3&4) further Material (Stage 4)	using and applying mathematics
Australia	Number;Space;Measurement;Chance and Data;Algebra	Attitudes and appreciation; Mathematical inquiry; Choosing and using mathematics
Singapore	**Primary School**: Whole Number;Money and Measures;Fractions;Decimals;Statistics;Geometry **Secondary School:** Arithmetic;Measurement;Algebra;Graphs;Statistics; Geometry;Trigonometry	
Hong Kong	**Primary School**: Number;Shape and Space;Measures;Data Handing;Algebra; **Secondary School**: Number & Algebra;Measures, Shape Space; Data Handling,	
Mainland China	**小學**： **量與計量；數與計算；幾何初步知識；統計初步知識；** **代數初步知識；應用題．** **初中：代數；幾何．**	
Taiwan	**小學**： **數與計算；量與實測；圖形與空間；統計圖表；** **數量關系；術語與符號．** **中學**： **數的概念；代數；平面幾何；坐標幾何；** **資料的整理與机率．**	
Korea	**Primary:** Number;Operation;Geometric figure;Measure- ment;Relation. **Middle:** Number and expression;Equation and inequality;Function¡ Q Statistics¡ Qgeometric figure;	

From tables 1 and 2, the following features of the Hong Kong mathematics curriculum are identified:

(1) In general, the mathematics curriculum in Hong Kong is in line with the worldwide trends. For instance,

- basic mathematical knowledge and skills are stressed (e.g. aim #3 in primary school, and aims #2, 3 in secondary school)
- mathematics “senses” and applications are emphasized (e.g. aim #4 in primary school and aim #1 in secondary)
- attitudes and affective factors for mathematics learning are taken into account (e.g. aims #1, 6 in primary school and aims #3, 4 in secondary school)
- emphasize process abilities such as problem solving, reasoning, and communications skills are emphasized (e.g. aim #2 in primary school and aim #1 in secondary school)

(2) The Impact of Information Technology on the Mathematics Curriculum

The Hong Kong secondary mathematics curriculum document (1998, p34) gives a detailed explanation on the impact of information technology on the mathematics curriculum, and makes some suggestions about how information technology can be used in mathematics teaching. This is in line with the world trend.

Although the impact of information technology is addressed in the Hong Kong mathematics curriculum document, nothing is said on when and how the calculator and computer should be used to assist mathematics learning. In contrast, in USA, UK, Australia, and Singapore, much detailed description about how to use IT is given. For instance, in NCTM (1998, p. 172), it is suggested that "students might examine the two quadrilaterals below by exploring the similarities and differences using dynamic geometry software"; and in the UK curriculum document (Department of Education, 1995, p. 16), it suggests using "computer to generate and transform graphic images and to solve problems".

(3) The Process of Learning

As in the case of IT in the Mathematics Curriculum, the Hong Kong secondary mathematics curriculum document also gives a detailed explanation about the process of learning. However, while process abilities are generally emphasized in the West, much attention is paid on basic skills in Asian countries. The Hong Kong mathematics curriculum does try to strike a balance between process and content, but we should proceed to investigate how the two can be better linked to each other. As rightly pointed out by various scholars (see Section 1 of this report), conceptual understanding and mathematical skills are not segregated (Cai, 1999; Wu, 1997; Wong, 1995; Wong & Wong, 1997).

3.2 Mathematics Contents: Depth and Coverage

When the mathematics curriculum is implemented across the years of schooling, we can consider the contents as a "flow" across grades: topics are introduced, continue for a time, and cease to receive attention. The TIMSS results show that there are considerable national variations in flow, milestones, emphasis, and performance expectation. As far as this report is concerned, we find:

(1) In Hong Kong, the introduction of topics into the curriculum is on average 2 years earlier than the international median (Schmidt, et al, 1997, p. 81).

(2) The textbooks in Hong Kong focus much of their attention on the performance expectations of "knowing” and "using routine procedures" (Schmidt, et al, 1997 , p.243).

(3) The performance of Hong Kong students is good at routine problem-solving but not so good at solving exploratory problems (Law & Leung, 1996).

(4) There are eight topic areas for which the introductory grade varied most across various countries. Instead of looking at all the TIMSS topics, it will be instructive to do a more in-depth comparison, in terms of coverage and depth of treatment, on these eight topics (table 3(a)). These eight topics are:

- exponents and orders of magnitude
- measurement: estimations and errors
- 2-D geometry: Coordinate Geometry
- 3-D Geometry
- patterns, relations and functions
- equations and formulas
- data representation and analysis
- uncertainty and probability

(Schmidt, et al, 1997, p. 70).

(5) In addition to the eight topics above, Algebra is considered an important area in mathematics, and so an in-depth comparison is also performed for this topic area (table 3(b)).

Topics in tables 3(a) and 3(b) are analyzed and compared according to the categories on the distribution of contents and performance expectation used by TIMSS. In this comparison, schooling is divided into four stages: junior primary, senior primary, junior secondary, and senior secondary. For each stage in each country, an entry in the table indicates what performance is expected of the students for that topic (+1 stands for Knowing, +2 = Using routine procedure, +3 = investigation and problem solving, +4 = mathematical reasoning, +5 = communication). For example, +1 in the cell for Units at stage 1 in USA means that students in junior primary schools in USA are expected to learn Units only up to the expectation of knowing (rather than reasoning for example).

Table 3(a). Distribution of contents and expectations in eight countries and regions

	stages	USA	UK	Australia	Singapore	Hong Kong	China	Taiwan	Korea
Units	stage 1	+1	+1	+1	+2	+1	+1	+1	+1
	stage 2			+2	+3	+1	+1	+1	+1
	stage 3								
	stage 4	+3~5							
Perimeter, Area, and volume	stage 1			+1	+2	+1(3)	+2(3)	+1	
	stage 2	+2,5		+3	+2	+3	+2	+2	+2(5)
	stage 3	+3	+3	+3	+3	+3			+2
	stage 4	+3~5		+3	+3,5				
Estimation and errors	stage 1		+1	+1	+1	+1			
	stage 2		+2	+2		+1			+1
	stage 3	+2	+3	+2		+4			
	stage 4			+4					
2-D Geometry: Coordinate Geometry	stage 1	+1							
	stage 2	+1	+1						
	stage 3	+2	+2	+1	+2	+1			
	stage 4			+2		+2	+2,3	+2,3	+2,3
2-D Geometry: Basics	stage 1	+1	+1	+1	+1	+1	+1	+1	+1
	stage 2	+1		+2	+2	+2	+1	+1	
	stage 3	+3,4	+3	+3,4	+2	+2	+3,4	+3,4	+2,4
	stage 4			+3,4					
2-D Geometry: Polygons and circles	stage 1				+1	+1	+1		+1
	stage 2	+1		+2	+2	+3	+1	+1	
	stage 3	+3,4	+2	+3,4	+2	+3,5	+3,4	+3,4	+3,4
	stage 4	+3~5				+3~5			
3-D Geometry	stage 1	+1	+1	+1		+1	+1	+1	
	stage 2	+1		+2	+1	+1	+1	+1	+1
	stage 3	+3	+2	+2					
	stage 4						+3,4	+3,4	
Transformation	stage 1	+1	+1	+1					+1
	stage 2	+1	+2	+2	+1	+1			
	stage 3	+3,4	+3	+2	+3,5	+1	+1		
	stage 4				+2				+1
Congruence and similarity	stage 1								
	stage 2	+1	+1	+1					+2
	stage 3	+2	+2	+3,4	+3	+2	+3,4	+3,4	+2
	stage 4								
Patterns,relations , and functions	stage 1	+1		+1	+1				+1
	stage 2	+1,5				+1			+2
	stage 3	+3	+3,5	+3,4	+3	+2	+3	+3	+2
	stage 4	+3~5		+3,4		3,5	+3	+3	+2
Equations and formulas	stage 1	+1		+1					+1
	stage 2	+1			+2	+1			+3
	stage 3	+3	+3,5	+2,3	+3,5	+3,5	+3	+3	
	stage 4	+3~5	+3,5	+3,4		+3,5	+2,4	+2,4	
Data representation And analysis	stage 1	+1,5		+1	+2	+1		+1	+1
	stage 2	+2,5	+2,5	+2,5	+2,5	+2	+2	+2	+2
	stage 3	+3,5	+3,5	+3,5	+3,5	+3	+3	+3	+2
	stage 4	+3~5	+3,4	+3,4		+3,5		+3	+2
Uncertainty And probability	stage 1	+1		+1					
	stage 2	+2,5	+2,5	+2,5		+1			+1
	stage 3	+2,5	+3	+3,5		+1			+2
	stage 4	+3~5		+3,4		+2	+2	+2	+2

+1~Knowing, +2~ Using routine procedure, +3~investigation and problem solving, +4~mathematical reasoning, +5~ communication
Stage1~ Junior Primay,Stage2~ Senior Primary, Stage 3~ Junior Secondary, Stage 4~ Senior Secondary.

Table 3(b). Distribution of contents and expectations in eight countries and regions

	stages	USA	UK	Australia	Singapore	Hong Kong	China	Taiwan	Korea
Whole Number Meaning, Operation and properties	stage 1	+2	+2,5	+1	+2	+2	+3	+2	+2
	stage 2	+3~5	+3,4	+3,5	+3	+3		+3	+3
	stage 3				+3,4				
	stage 4								
Common fraction and their properties	stage 1	+1	+1	+1	+2,	+1	+1	+1	+1
	stage 2	+2		+2,5	+3	+2	+3	+3	+3
	stage 3	+3~5	+3~5	+3,5	+3,4			+3,4	
	stage 4								
Decimal fraction , properties, relationship of common and decimal fractions	stage 1		+1	+1		+1		+1	+1
	stage 2	+2	+2	+3	+3	+3	+3	+3	+3
	stage 3	+3~5	+3~5	+3,5	+3,4			+3,4	
	stage 4								
Ratio, Proportion,and Percentages	stage 1					+1			
	stage 2	+2		+2,5	+2	+3	+2		
	stage 3	+3~5	+3~5	+3,5	+3	+3		+2	
	stage 4								
Negative number, integers,and their properties	stage 1								
	stage 2		+1	+1		+1			
	stage 3	+3~5	+3~5	+2,5	+2	+2	+2	+2	+2
	stage 4							+2	
Rational Numbers and their properties	stage 1								
	stage 2	+1							
	stage 3	+3~5	+3	+3,4	+2	+2	+2		+2
	stage 4							+2	
Real number, their subsets and their properties	stage 1								
	stage 2								
	stage 3	+3	+3		+2	+2	+2		+3,4
	stage 4			+3~5				+2	
Exponents, roots, and radicals	stage 1								
	stage 2								
	stage 3	+2	+2	+2	+2	+2	+2	+2	+2
	stage 4								
Number theory (factor, multiples,primes etc.)	stage 1								
	stage 2		+2	+2	+2	+2	+2	+2	
	stage 3	+2		+2					+2
	stage 4	+3							
Estimation and number sense	stage 1	+1		+1		+1		+1	
	stage 2	+2	+2	+3	+2	+2		+2	
	stage 3	+3~5	+3~5	+3~5	+3,4	+3		+3	+2
	stage 4			+3~5					
Slope and trigonometry	stage 1								
	stage 2								
	stage 3	+3	+3	+3	+3	+3	+3		+3
	stage 4			+3,5		+3~5		+3	

+1~Knowing, +2~ Using routine procedure, +3~investigation and problem solving, +4~mathematical reasoning, +5~ communication
Stage1~ Junior Primay,Stage2~ Senior Primary, Stage 3~ Junior Secondary, Stage 4~ Senior Secondary.

From the summarized data in table 3(a), and from the curriculum original documents, we can see that:

(1) Asian countries and regions put a lot of emphasis on measurement (units, perimeter, area & volume, estimation and errors), much more so than West countries. In particular, Singapore stresses measurement units while Hong Kong stresses estimation and errors more than the other countries in Asia.

(2) Different approaches to Coordinate Geometry

In Hong Kong, Coordinate Geometry is introduced in junior secondary school, and it is taken as a

connection between Geometry and Algebra. However, the coordinate system is introduced in the primary years in USA and UK, and the approach is very different. Fung & Wong (1997) suggested that coordinates taken as an algebraic treatment of Geometry should be down-played, but using it to describe location should be emphasized.
As for " classical" Coordinate Geometry such as equations of ellipse, hyperbola, and parabola and their properties, it is only introduced in China, Taiwan and Korea.

(3) Except for China and Taiwan, all other regions do not introduce classical Euclidean Geometry using an axiomatic approach. Geometry is regarded as a means to develop ability in problem-solving, reasoning, and spatial sense among students. In Hong Kong, reasoning through Geometric problems is only introduced in Grade 10, which may be a bit too late.

(4) Transformation, Congruence and Similarity are regarded as good topics for developing deductive reasoning and the idea of proof, and in many countries they are introduced in junior secondary school. With the assistance of interactive computer software such as Geometer's Sketchpad, it may be possible for these topics to be introduced at an earlier age. This seems to be the practice in USA, England, Australia, and Korea, where these topics are first introduced in primary school.

(5) Similarly, Number Patterns and Relations are good topics for enhancing students' abilities in exploring , conjecturing and reasoning, and we should consider introducing them in early years such as in the cases of Korea, Singapore, and USA.

(6) Statistics and Probability are very useful knowledge in the information society. In Hong Kong, data handing was emphasized all through the school years. However, more emphasis should be put on the interpretation and analysis of data, as in other countries such as Singapore, Australia, England and USA. Probability should also be introduced from the early years.

Data in table 3(b), supplemented by the original curriculum documents, give similar results to those found in TIMSS (Schmidt, 1997, p17-19) concerning Number and Operation. In brief, the following observations can be made from the data:

(1) In Hong Kong, Mainland China, and Korea, Fraction and Decimal are introduced, dealt with thoroughly and completed in primary school, whereas in other countries, these topics are continued to be dealt with in secondary school. This shows that Hong Kong, Mainland China and Korea stress computation ability in early grades.

(2) Introducing Negative number in primary school is uncommon in East Asian mathematics curricula. The trend in other countries however, is to introduce Negative Numbers in early years, since negative numbers is frequently met in daily life.

(3) Hong Kong is one of those few places (together with Singapore, Mainland China and Taiwan) that introduce a variety of number theory topics such as factors, prime number, HCF& LCM etc. There may be a need of trimming down this part of the curriculum, and the new curriculum in Hong Kong seems to be moving in this direction (The Curriculum Development Council, 1997).

(4) Estimation is stressed in Hong Kong, and this is in line with the international trend of reform in the mathematics curriculum.

3.3 The Tracking of Mathematics Curriculum Development

Compulsory education is an important and challenging issue confronting all countries. On the one hand, we should go for a "mathematics for all" curriculum, and on the other hand, mathematics teaching should cater for individual needs. Different countries have carried out different mathematics curriculum reforms to cater for the varied needs of individuals, and these reforms are designed according to their own traditions and cultures. Some of these efforts are portrayed below.

3.3.1 Australia

3.3.1.1 Curriculum Decision Making

School education is the responsibility of the individual states and territories, although the influence of the federal government has grown in recent years. There is no national curriculum for mathematics, but the *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1990, 1995) provides an indication of the general thinking about mathematics curricula in Australia. Different provinces such as Victoria also produces their own curriculum standards. A significant development in 1990 was the establishment of the Curriculum Development Corporation by the state and federal education ministers. This is a semi-autonomous body with a charter to develop curriculum materials on a commercial basis.

3.3.1.2 Tracking and Textbooks

From Grade 1 to Grade 9, there is almost no systematic grouping and most classes are heterogeneous in their composition. In grade 10, 11, and 12, it is common for mathematics to be provided at different levels and for students to choose the level appropriate to their future plans.
Over the past 10 years, there has been a reduction in the emphasis placed on the use of textbook. Textbooks are generally used under teacher guidance as a source of activities for the reinforcement of concepts and for practising skills, and textbooks are not developed or prescribed by the state departments of education. Decisions about textbooks, and the extent of their use by students, are made at the school level.

3.3.2 Mainland China

3.3.2.1 Curriculum Decision Making

In the past, the syllabus is centrally devised in committees set up under National Education Council (NEC), and textbooks were produced solely by the People's Education Press. But since 1992, under the so-called "One syllabus, many textbooks" policy, eight provinces have been authorized to produce their own textbooks.. This later evolved into a "multiple syllabuses, many textbooks" policy with places such as Shanghai having their own syllabuses and textbooks.

In Shanghai, the mathematics reform has recently gone through another tremendous innovation with the following features:

- regarding the 9 years of compulsory schooling as a whole period as far as the curriculum is concerned
- the syllabus is presented in the language of Bloom's taxonomy
- unifying Arithmetic and Algebra;
- Geometry is developed from visual, operational, to reasoning
- the framework consists of three parts: a compulsory course, some selective courses, and activities

At the same time, the People's Education Press is striving to develop new textbooks catering to the demands of compulsory education. In addition, it is worth paying attention to *The Agenda for Action for School Mathematics Education Toward the 21st Century (draft)* in Shanghai, in which the enhancement of students' ability is taken as the major task. The National Education Council is also drafting the National Mathematics Standard Towards the New Era (1999).

3.3.2.2 Tracking and Textbooks

As mentioned above, since 1992, eight provinces have been authorized to produce their own textbooks. Table 4 shows the different tracks in the mathematics curriculum.

Table 4 tracking of the mathematics curriculum (National syllabus)

Grade 1 to 6	Grade 7 to 9	Grade 10 to 12
Primary mathematics	mathematics (basic- graduate examination) (selective- high school examination)	Mathematics (compulsory) Mathematics (science stream) Mathematics (arts stream) Mathematics (vocation stream)

3.3.3 Korea

3.3.3.1 Curriculum Decision Making

In South Korea, there are three tiers of administration in the education system: the Ministry of Education, offices of education at the provincial level, and local offices of education at the county level. The Ministry of Education is responsible for forming polices relating to education, publishing and approving textbooks, directing subordinate agencies in planning and policy implementation, and providing financial support for nation universities.

Korea's schools are required to follow a national curriculum set by the Ministry of Education. School mathematics curriculum of modern Korea has been revised six time since the establishment of it government in 1945.

3.3.3.2 Tracking and Textbooks

There is no official policy on within-school tracking in South Korea, and primary and secondary school classes are composed of students with a wide range of abilities. All students study a compulsory program of mathematics up to the end of Grade 11. General mathematics is usually offered up to Grade 10, while in Grades 11 and 12, elective course are offered for Science, Humanities, and Vocational majors (see Table 5).

Table 5 The tracking in mathematics course in Grades 1 to 12

	Grades 1 to 10	Grade 11 and 12	Tracking
6^{th} Mathematics courses	common	Mathematics I Mathematics I,II Practical Mathematics	Literary Track Science Track Vocational
7^{th} Mathematics courses	Mathematics Level 1 Mathematics Level 2 (Basic-common, Enrichment, Supplement)	Mathematics I Practical Mathematics Mathematics I,II Differentiation and Integration, Probability and Statistics, and Discrete Mathematics	Literary Track Vocational Science (Natural Science or Technological) Track Science (Natural Science or Technological) Track Vocational (With having to have completed the 10th level Mathematics) Regardless they have or have not successfully completed the 10th level mathematics.

In elementary schools, students use textbooks and workbooks published by the Ministry of Education for each grade level. In 1987, the policy on secondary school textbooks was changed from a government authorized to a government approved system. Secondary school mathematics textbooks are usually developed by university professors with the participation of mathematics teachers.

3.3.4 Singapore

3.3.4.1 Governance and Decision Making

The Ministry of Education consists of a Ministerial committee and 10 divisions. The Ministry's role is to develop national education goals and a coordinated education program for the whole country. Responsibilities for curriculum development, textbooks selection, instruction, and examination standards are centralized in the Ministry.

3.3.4.2 Tracking and Textbooks

Singapore provides a multiple curriculum which is flexible. Students follow different curricula according their abilities and potentials. Primary schooling is divided into two stages: Foundation stage (grades 1 to 4) and Orientation stage (Grades 5 to 6). There are four kinds of courses at the Orientation stage:

- EM1 (suitable for students who have ability in academic subjects and language);
- EM2 (suitable for students who will study English and the native language);
- EM3 (suitable for students who have lower ability and will study English and "basic" native language); and
- EM4 (suitable for students who will study advanced native language and basic English.

There are four courses in the secondary school curriculum:

- Special course (four years, for the top 10% of graduates from primary school; G.C.E. A. Level);
- Express course (four years; students will learn English and the native language; G.C.E. O. Level);
- Normal (Academic) course (four or five years; students after 4-years may take the G.C.E.' N' Level, those who perform well can continue for one more year and take G.C.E. ' O' Level);and
- Normal (Technical) course (four or five years; students after 4-years may take the G.C.E.' N' Level, those who perform well can continue for one more year and take G.C.E. ' O' Level).
- G.C.E ' N' Level, the top performance students can continue to learn one years, G.C.E. ' O' Level)

Table 6 Tracking of mathematics curriculum

Grades 1 to 4	Grades 5 to 6	Grade 7 to 10
Common	EM1/EM2 EM3/EM4	Special/Express Normal academic/ Normal technical

The Ministry of Education provides a list of approved textbooks and instructional materials to assist principals, department heads, senior subject teachers, and subject coordinators in selecting suitable textbooks for their pupils. These approved textbooks include those commercially produced as well as those published by the Curriculum Development Institute of Singapore, which is part of the Ministry of Education. Textbooks normally follow the intended syllabus very closely. All students have their own mathematics textbooks, which are used mainly for teaching, as well as for review and assignments.

3.3.5 United Kingdom

3.3.5.1 Curriculum Decision Making

The Education Act (1988) established a national curriculum and national curriculum assessment in England. The central government has the authority and responsibility for the provision of education services, for determining national policies, and for planning the direction of the system as a whole. At the local level, policies are implemented by local education authorities and school governing bodies, together with further and higher education institutions. It is the responsibility of the school governing bodies to allocate the budget and to determine the general direction of the school and its curriculum, subject to the requirements of the national curriculum.
The Mathematics in the National Curriculum (Department for Education, 1995) only states the Programmes of Study and the Attainment Targets, and does not set rigid schedule of teaching nor time allocation for teaching different topics. So there are enough flexibility for schools to make some choices according to their needs.

The Programmes of Study presents the curriculum in four content areas (Number and Algebra, Shape, Space and Measures, Handling Data) and one process dimension (Using and Applying Mathematics) over various stages, to be checked across ten levels of attainments. The relation between grades and attainments is listed as follows:

Table 7 the attainment level over stages

Stages	Year group	Grades	Attainment Levels
Stage 1	5-7	1-2	1-3
Stage 2	7-11	3-6	2-6
Stage 3	11-14	7-9	3-8
Stage 4	14-16	10-11	4-10

3.3.5.2 Tracking and Textbooks

There is no official policy on within-school streaming, and most schools are not streamed. Primary school classes generally include students of varying abilities. Mathematics form parts of the curriculum for all students of compulsory school age. Streaming is quite common for secondary mathematics classes.

Textbooks are normally commercially produced. A vast choice of textbooks exists in England at both the primary and secondary levels. One of most widely used series of the secondary textbooks is the *School Mathematics Project* series, which was first developed in the 1960s.

3.3.6 USA

3.3.6.1 Curriculum Decision Making

In general, education is a responsibility of individual states. However, all states are required to operate schools meeting federal standards. The United States Department of Education, with the participation of more than 30 federal agencies, is responsible for federal education policy. The majority of the states also develop their own curriculum guidelines, at least in the major academic areas, and professional standards for teachers. United States first launched the mathematics standards in 1989 with the publications of NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989*), Professional Standards for Teaching Mathematics* (1991*), and* the *Assessment Standards for School Mathematics* (1995), together with the addenda series. Thereafter, some states drafted their own standards too, e.g. the California Mathematics Framework and some others. The idea was supported by the government and the president explicitly mentioned about the standards in his State of Union Address in 1998. At present, the NCTM *Standards* is under revision and the *Principles and Standards for School Mathematics* (draft)(1998) has just been released for consultation. In the near future, the mathematics curriculum will be influenced in significant ways by an ongoing federal project initiated in response to what has been perceived as the unacceptably low international standing of U.S students' achievement, and the concern this raises for the future global economic position of the United States.

3.3.6.2 Tracking and Textbooks

Grouping by ability occurs in many schools, although most do not support overt tracking by ability. Students with higher grades may specialize in mathematics and science early, increasing the number of optional, specialized mathematics and science courses as they reach higher levels.
Textbooks are produced by private corporations, with very few such corporations dominating the market. A number of states recommend textbooks for local use after some process of review against state curriculum guidelines. In all cases, specific textbooks selection is a local decision, although this selection may be limited to those approved by the state.

3.3.7 Conclusion

- East Asian countries have highly centralized systems of education but Western countries have much more decentralized systems.
- A "canonical" curriculum is stipulated by the governments in Asian countries which are followed closely in schools.
- East Asian countries put a lot of emphasis on the textbooks; in contrast Western countries are more flexible in their use of textbooks.
- Tracking is common, but there are various ways implementing tracking
- Hong Kong is probably the place with the least flexibility and choice in its curriculum.

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