11. RECOMMENDATIONS

The research has been successfully completed, yielding fruitful results. From the rich data collected, a clear holistic picture of the Hong Kong mathematics curriculum has emerged. Such a large scale situational appraisal has never taken place before. It should serve as an exemplar to other subject areas as well as to societies in neighbouring regions. The data collected and the delineation thus obtained should be of great significance to subsequent mathematics curriculum reform.

In general, views of various sectors on the mathematics curriculum have been positive. Students and parents showed a high regard for mathematics. Teachers and parents reflected that the existing curriculum has given students a strong grounding in basic skills which partially explains why Hong Kong students have achieved outstanding results in international comparisons on mathematics performance. Teachers also pointed out that the current curriculum documents are clear and easy to follow. Parents showed full support for both the curriculum and their own children's learning. End-users such as human resources personnel in commercial and business enterprise and university lecturers were generally satisfied with the educational output. By and large, different departments in universities were able to recruit undergraduate students of a suitable mathematical calibre. Employers viewed public examinations as reliable indicators of ability. The curriculum planner from the science area also reflected that the mathematics curriculum can adequately support students' learning in the science discipline. In all, this positive attitude and support from the various sectors involved in the study is conducive to the notion of mathematics curriculum reform.

Certainly, it does not mean that the current mathematics curriculum has no room for further improvements. With the implementation of universal education, mathematics education needs to accommodate a wider spectrum of students ability, hence, it should carry a different set of objectives. In our highly technological society, jobs have now changed from skill orientations to processing of information. Consequently, the ways in which mathematics is learned are now far more important than what is learned. Thus the mathematics curriculum needs to be reformed to meet the challenges of the new millenium. The views of the various sectors collected in the present research are invaluable for curriculum experts to design the future mathematics curriculum.

Here, we will first summarise the findings of the study, and then attempt to provide a holistic picture of the situation. Following that, we shall make recommendations based on the research findings to point out the direction of mathematics curriculum reform.

11.1. Summary of findings

Review of literature revealed that individual differences and mixed abilities are major issues of concern for mathematics education in the next century. Flexibility of the curriculum is asked for to cater for the above. In a pluralistic and highly technological society, mathematics should be taught as a subject which possesses several very different goals that reflect the diverse roles mathematics plays in the society. Mathematical knowledge, concepts, problem solving skills as well as abilities to discover and to invent should be encompassed. The development of assessment of higher order thinking is a world issue but we should, at the same time, safeguard against having the curriculum driven by examinations. Both the "content" and "process" of mathematics learning have to be taken care of. Cultivation of interest in learning mathematics is important too. In fact, international comparisons revealed that, though Hong Kong students performed well in mathematics tests, they lacked confidence in solving mathematical problems. How and when information technology could be used to enhance mathematics learning is another important issue that is urgently needed to be explored. In order to leave room for cultivating the motivation of student learning and the development of higher order abilities, the scope and depth of the current curriculum content have to be re-considered. Again, international comparisons revealed that in Hong Kong, on the average, topics were taught one or two years earlier than in other countries. All these new ideas cannot be accomplished without the professionalism of mathematics teachers. Previous research did reveal student learning in mathematics was greatly influenced by the teacher.

By the use of student questionnaires, we see that students possessed a high regard for mathematics and preferred deep understanding rather than rote memorisation. They wished to know how formulas come about and are applied. They found interest in learning mathematics at a young age, though such an interest declined and they found mathematics learning more and more difficult at higher grade levels. They experienced the greatest pressure from homework at Primary 6. Topics that involved tedious calculations were least welcome and word problems were thought to be difficult. Students hoped for liveliness and real life applications both in teaching and in textbooks. Secondary school students felt that the syllabus at the

junior secondary level was too fragmented and there was much overlapping of topics at Secondary 1 with those at primary levels. Senior secondary school students showed dissatisfaction with the whole senior secondary and sixth-form mathematics curriculum structure. They reflected that the syllabuses could not cater for their needs.

Interviews with students further reinforced these findings. Students generally saw mathematics as a set of rules. At the same time, they realised that the way one approaches a question and applies a formula, and even one's way of thinking are important if one is to solve mathematical problems. They saw homework as an important component of mathematical learning and so they hoped that teachers can provide them with sufficient exercises that provoke thinking. Their image of a good mathematics teacher was someone who is nice, lively, provides a variety of activities, who offers clear, step-by-step explanation, who allows time for students to think, checks frequently to see if students understand from time to time, explains how to approach problems and would not penalise weaker students. They reflected that their interest in learning mathematics was closely related to whether they could obtain a sense of success in solving mathematical problems. As found in the questionnaires, students disliked topics that involved tedious calculations, or were easy to make mistakes, impractical or difficult. Besides those discontents on the curriculum found in the questionnaires, students further pointed out that the curriculum was too packed in general and the case was even more serious at the senior primary level due to over-drilling for the Academic Aptitude Test. Some students found the use of computer software in teaching mathematics a waste of time.

As reflected in the parent questionnaire, parents showed high regard for mathematics and held a positive view towards the mathematics curriculum. Students' interest and understanding were their sole concern. Consistent with what was found among the students, parents hoped for clear explanation, motivation of interest, provision of exercises that provoke thinking and checking of students' understanding from time to time by the teacher. Parents showed great support to their children's learning of mathematics. They were willing to help with their children's learning and many of them, especially parents of students at lower grade levels, employed private tutors for their children. They believed in paying effort and relied on practices. To them, the major problems among students were carelessness and inability to interpret the questions. As their children moved up the grade levels, parents possessed less knowledge of the

curriculum and there was a tendency of relying more on traditional ways of learning, like drilling with exercises. Parents generally had a negative feeling towards the Academic Aptitude Test and the quality of mathematics textbooks.

Interviews with university lecturers revealed that they were generally satisfied with students' standard and curriculum and saw the scores in public examinations as reliable. The demand on mathematics varied across departments and basically they could admit students of the appropriate calibre through competition of examination results. The only possible exception was the department of mathematics where they hoped to get students with a strong mathematical foundation and in reality this was not always possible. Some lecturers asked for breath and some asked for depth in the school mathematics curriculum, but in general, a firm foundation and mathematical sense were of utmost importance. However, most of them did not possess much idea of the existing mathematics curriculum nor what was currently going on in school mathematics.

Most of the employers were satisfied with students' performance too. They saw language and attitude more important than mathematical knowledge. However, analytical power, problem-solving skill and a sense of numbers were important in most of the careers.

The curriculum planner in the science area was satisfied with the mathematics curriculum, saying that it could provide the science subjects with necessary mathematical tools. Interviews with mathematics curriculum planners at various levels revealed various problems of the existing mathematics curriculum. Lack of continuation from kindergarten to primary mathematics, inability to cater for individual differences at Certificate level, curriculum being too packed in general, both Pure Mathematics and Applied Mathematics being too difficult were some of the problems they raised. Some suggested a shift from computation to conceptual understanding in the mathematics curriculum. They showed discontent about the over-emphasis on examinations in the community. The curriculum planners also urged for more communication between primary and secondary school teachers to improve coordination between these two levels.

As reflected from the teacher questionnaire, mathematics teachers felt they possess adequate mathematics knowledge to teach, except for calculus and classical mechanics in the sixth-form where some teachers did not have sufficient confidence. Ability and motivation to learn were perceived as the major problems among the

students in learning mathematics. Mixed ability was another key issue too. The students performed less well in those topics that involved tedious computation. As for the curriculum, most of the mathematics teachers who participated in the survey reflected that it was too bulky, lacked flexibility, was unable to cater for individual difference and to provoke thinking. The content was found to be dry too. Teachers had a tendency to tackle individual differences themselves and were not inclined to more systematic ways such as setting different assessment standards for different classes since fairness was an issue of concern. Not many of the teacher incorporated information technology in their teaching at the moment and when they wanted to seek help in their teaching, collegiate exchange, their own school experience and textbooks would be their preferred means. They seldom took the curriculum documents or seminars as a source of help. It is note-worthy that many primary school teachers did not have strong mathematics background and they generally urged for a reduction of workload, in particular of non-teaching duties.

The above is consistent with what was found in the interviews with teachers in which they said that students were good at mechanical computation but weak in conceptual understanding and higher order thinking. Students had a short attention span, and at the secondary level, students were perceived to be passive, unable to take the initiative and not serious about learning. Another serious problem was that they lacked a solid foundation. Almost all teachers pointed out that the existing mathematics curricula were too packed, too boring, impractical and unrelated to real life. They advised that continuity at all levels must be secured. Contents and level of difficulty should be rearranged with a strong epistemological and pedagogical foundation. If streaming is to take place at the senior secondary level, then opportunity for further mathematics studies at sixth-form must be offered as a viable option. The idea of a core and extended curriculum seems to be workable but we must let parents understand the rationale behind this notion. Teachers agreed that higher order abilities should be addressed. The curriculum should be trimmed down to leave time and space for this to take place. Teachers generally showed high regard for information technology but they lacked guidance and support at all levels. Furthermore, they considered the use of information technology to be time-consuming. All in all, time is a big concern for teachers. Teachers need more time to prepare teaching materials. Suggestions from teachers include reduction in teacher-student ratio, class size and teaching workload, improvements to the crowded workplace, teachers' morale and social recognition of their profession.

11.2. Conclusions

The above results gave a clear picture that the current mathematics curriculum was well supported by various stake-holders though there are rooms for improvements. They cast great trust on the existing system. Both students and parents showed high regard for mathematics and they all opted for understanding rather than learning by rote. These are advantageous factors for the mathematics curriculum reform. Students, parents and teachers saw the basic skills as some of the strengths of the current curriculum and it can provide the students with a solid foundation. This is also reflected in the results in international comparative studies. Teachers also found the curriculum clear and easy to follow. We think that such strong points should be retained in the future mathematics curriculum innovation. We repeat our view that there is room for improvement if we are to develop a high quality mathematics curriculum for the new millenium.

Mathematics education should possess an expanding goal that reflects the diverse roles mathematics is expected to play in society. To maintain the interest of learning mathematics among our students, mathematics should be taught in a more lively way. So, textbooks should build in a variety of learning activities including real life examples and exercises that provoke thinking. The position and use of information technology in mathematics education is an issue of concern among different stake-holders but we need further developmental research to explore how and when information technology can be used to make mathematics learning more effective.

Individual differences among the students is a major issue in the period of universal education. To cater for such needs, curriculum differentiation has to be considered which includes the reorganisation of the senior secondary and sixth-form curriculum structure. The idea of core and extended curriculum worths further exploration. In order to address process abilities, the curriculum should be enhanced to ensure continuation at different levels, from kindergarten to sixth-forms, and to avoid overlapping and fragmentation. It is timely especially when the language barrier at S.1 is basically removed for most schools. Unnecessary mechanical calculation and impractical topics should be removed. If there should be a trim-down of the contents, it is only because we want to spare room for deeper understanding of the material rather than going for a water-downed curriculum. We need strong theoretical foundation to reorganise the content of the mathematics reasonably.

No curriculum can be successfully implemented without the teacher. We advise that teachers should play an active role in the new curriculum. We are of the view that the curriculum should not be a document that is passed to the teachers to follow. A new curriculum will impose new demands on mathematics teachers. Teachers are expected to teach in a more lively manner to develop students' interest and confidence in doing mathematics and to give them a sense of success. Teachers should also possess the ability to handle individual differences. They should aim at encouraging higher order thinking in students and helping them develop problem solving abilities. Mathematics teachers should incorporate sensible information technology in their teaching to make mathematics learning more effective. Teachers' conception of mathematics and of mathematics learning should be widened too, possibly through wider exposure that can enrich their mathematical experiences.

All these cannot be done without the enhancement of teacher professionalism. Teacher training and support are important. Teachers would need guidance on various issues like curriculum tailoring and the use of information technology. Peripheral support should be strengthened, and teacher workload should also be seriously re-considered. Collegiate exchange among mathematics teacher both within schools and in the wider mathematics education circle should be encouraged. In particular, communication among mathematics teachers at primary and secondary levels should be strengthened. The Academic Aptitude Test is found by many to be disruptive. This is a sure sign that there is an urgent need for review. The significance of higher order abilities should be reflected in assessments but every effort should be made to guard against any backwash effects. We realise that reliable testing items of higher order abilities have not been fully developed any where in the world. Therefore we should proceed with caution and research carefully on this area before the actual implementation of the idea. Different stake-holders, in particular university lecturers and parents, should be fully informed of the aim and direction of the new curriculum. Finally it is important to solicit the support of parents for the curriculum, and this, in turn, would ensure that they continue to encourage and assist in children's mathematics learning. In the end, it is the students who will reap the benefits of a sound mathematics education.

11.3. Recommendations

11.3.1. Curriculum structure

As seen from the above, the urge for restructuring the mathematics curriculum

structure at senior secondary level is clear. Its major purpose is to cater for individual differences. In particular, there will be students using mathematics to very different extent after their school mathematics studies. There are those who will go for further studies in the disciplines of science, commerce and social sciences. There are also those who will join the working force in different walks of life. The curriculum structure at senior secondary level has to be considered together with that at the sixth-form level. Such a need has been repeatedly voiced out by teachers and students too.

First, we think it unnecessary to have curriculum differentiation at the years of compulsory education (P.1 to S.3). It is appropriate to have a single "mathematics-for-all" syllabus and we shall call it, for the time being, "General Mathematics".

At present, some ten thousand students sit for CE Mathematics each year. A single syllabus (we refer to teaching syllabus rather than examination syllabus unless otherwise stated) cannot cater for the needs of such a large population. In addition, having three syllabuses at Certificate level is impractical for a number of reasons, though we do not exclude such a possibility. It is most likely that the number of candidates in one of the syllabuses will be very small if we have three syllabuses. Besides, for obvious administrative reasons, each school would only offer at most two syllabuses and take "class" as a unit. We shall elaborate our design based on the conclusion of having two syllabuses here.

There are three alternative curriculum models, viz., (1) Mathematics 1/Mathematics 2, (2a) Mathematics for the whole population and Additional Mathematics for a subgroup with the content of the latter closely related to those at the sixth-form level, (the current situation) and (2b) Mathematics for the whole population and Additional Mathematics for a subgroup with the content of Additional Mathematics detached from those of the sixth-form syllabuses.

On further analysis, if the content of Additional Mathematics is not related to sixth-form mathematics (i.e. (2b)), probably only a few schools would offer such a syllabus, and if it is linked to sixth-form mathematics (i.e. (2a)), it will attract those students not well prepared (incompetent) to take that subject as they do not want to miss the chance of further study in the mathematics area in S.6. In any case, the general mathematics course still has to face the ten thousand students per year, which is not a desirable situation. In sum, the Mathematics/Additional

Mathematics model cannot solve the problem we are currently facing and we should seek reconstructing the mathematics syllabuses under the Mathematics 1/Mathematics 2 model.

So we recommend to adopt the Mathematics 1/Mathematics 2 model with the latter syllabus aiming at preparing students for further studies in mathematics up to A Level and the other targeted to those who would not continue their mathematics studies in sixth-form or those who intend to do either mathematics or statistics courses at AS Level (we deliberately use lower case to indicate that we are not referring to the existing AS Mathematics and Statistics syllabus). We would not block the possibility of Mathematics 1 proceeding to AS Mathematics and Statistics nor Mathematics 2 proceeding to AL Mathematics. The design of the curriculum structure are based on the following principles:

(a) Mathematics 1 will gear to those students who will not need much mathematics after Certificate level, while Mathematics 2 will cater for students who want to do more mathematics after Certificate level.
(b) Both syllabuses should be suitable for those who will not continue their mathematics studies after graduation at Certificate level.
(c) We would not block the chance for those who studied Mathematics 1 to study some mathematics (at least the equivalence of current AS Mathematics and Statistics) at the sixth-form level.

We expect that most Arts students would take Mathematics 1, but still there would be some science students taking it. Next we will describe the linkage of these two syllabuses with sixth-form mathematics.

For the Mathematics 1 track, two independent AS syllabuses, namely AS Mathematics and AS Statistics will be designed for those who wish to go for further studies in mathematics in the sixth-form. Both occupy half of the normal teaching time and will be studied in S.6 through S.7.

As for the Mathematics 2 track, three AS syllabuses, namely AS Foundation Mathematics, AS Applied Mathematics and AS Further Mathematics will be designed. The first is the prerequisite to the other two, though AS Foundation Mathematics can be taken independently. The examination syllabuses for AS Mathematics and AS Foundation Mathematics will be the same. Of course the approaches to them will be geared to the orientation and characteristics of the

student populations of each course. Foundation Mathematics will be offered in S.6 whereas the other two courses will be offered in S.7 or S.6-7.

Of course, students who take Mathematics 2 at S.4-5 can choose to take AS Mathematics and/or AS Statistic in S.6-7. At the same time, we do not block those who have taken Mathematics 1 at S.4-5 to take AS Foundation Mathematics (and AS Applied Mathematics/AS Further Mathematics) at S.6-7 if they can show a good performance in Mathematics 1. The structure of the recommended S.4 to S.7 mathematics curriculum is illustrated in Figure 50.

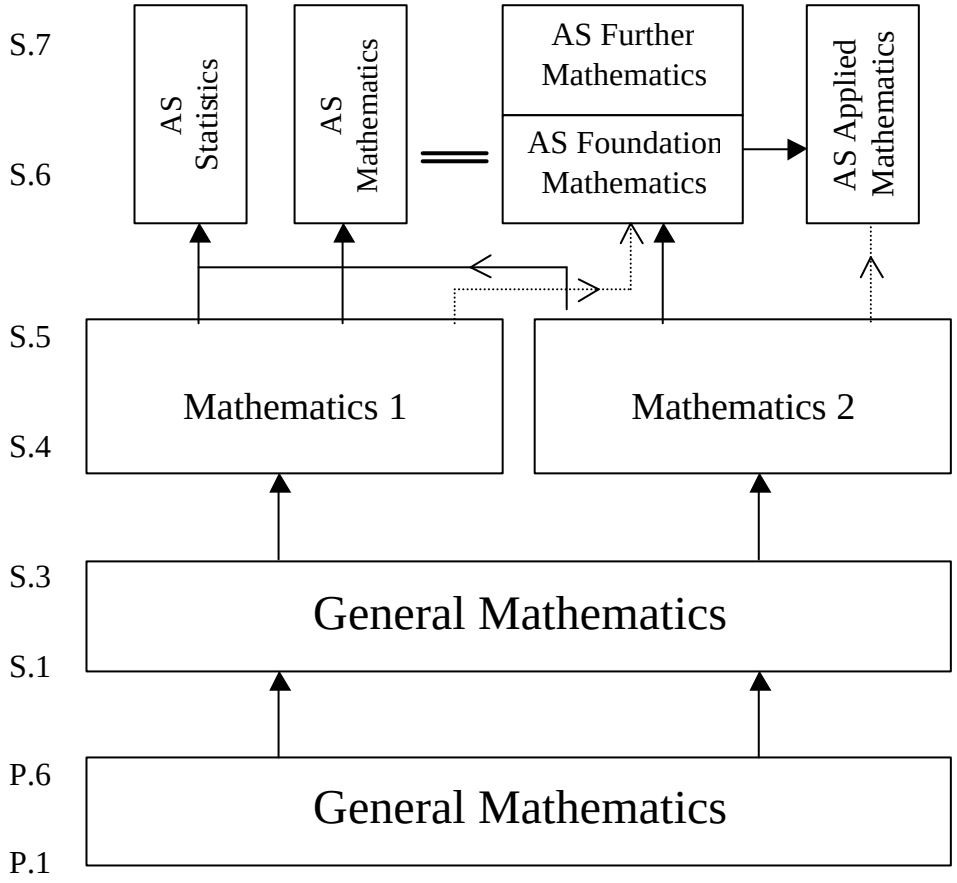

Fig. 50 Proposed structure of the S.4-S.7 mathematics curriculum

11.3.2. Principles of curriculum design

In order to secure an appropriate progress of General Mathematics in the years of basic education and to ensure continuation at different key stages of learning, we need strong epistemological and pedagogical considerations in designing the curriculum. Ideas in Fung & Wong (1997) might be helpful and we strongly suggest that future curriculum designers should create a flow chart of topics from P.1 to S.3 before putting in specific topics at different levels. We hope that the

final P.1 to S.3 curriculum document would be presented in a single volume (together with primary and secondary mathematics put in different booklets). In order to enhance the curriculum and to put in ingredients of information technology and higher order thinking, the present curriculum should be reorganised. Probably some topics have to be removed so as to leave space for the above. Again, epistemological and pedagogical considerations are essential to safeguard against ending up with a fragmented curriculum.

We do take curriculum tailoring as one of the effective ways to address individual differences and we agree that the identification of "core part" of the curriculum in the way of treatment as proposed by the *Tailoring Guide for the Secondary School Curriculum* (CDC, 1996) (i.e. not going for separate examination papers) could downplay labeling effect. When the new General Mathematics is designed with an epistemological framework, the identification of a "core part" should come up in a more reasonable way. The effect of curriculum tailoring by such means needs further research. Two areas requiring further investigation would be ways to ensure successful implementation and the peripheral support needed for success.

At the senior secondary (S.4-5) level, the foci of Mathematics 1 and Mathematics 2 are clear from the respective target student population, and clearly, Mathematics 2 is a stepping stone to sixth-form mathematics. As for Mathematics 1, besides introducing more practical topics, some of the S.1-3 General Mathematics can be revisited.

Calculus in the current Additional Mathematics syllabus will go into the proposed AS Foundation Mathematics, which will also include "system of linear equations". Numerical analysis and differentiation will be the main themes of AS Applied Mathematics and it will not contain classical mechanics. As for AS Further Mathematics, it will include the topics of conic curves, further calculus, set language, vectors, three-dimensional geometry, matrices, complex numbers, equation and polynomials. How much this syllabus will contain is subject to further investigation but the following should be taken into consideration. First, only mathematically inclined will be encouraged to take this course. Second, schools will not offer this course independently but will only offer together with AS Foundation Mathematics. So in terms of teaching time, teachers would probably start teaching topics in AS Further Mathematics at Secondary 6.

11.3.3. Implementation strategies

The Curriculum Development Institute should develop and issue clear guidelines and exemplar materials for teachers, textbook publishers and textbook writers so that they would be able to develop sufficient materials suitable for their users. We also need a group of high calibre professionals working full-time in the central agency to provide strong supports and guidance for school teachers to tailor-make teaching strategies for their students.

Communication with different stakeholders is essential to ensure the successful implementation of the new curriculum so that they understand the need and orientation of the change. The notion of minimal competence by criterion referencing at different key stages can be explored.

11.3.4. Assessment

Assessment of the new curriculum will have to be conducted in line with the design and direction of the new curriculum so that feedback of student learning can be effectively collected. At the same time, every precaution should be taken to safeguard against the curriculum being driven entirely by examinations. In particular, it is high time for the Aptitude Academic Test to be reviewed as it has found to be disruptive by all parties. We recognise that there is a need for assessment of higher order thinking, but reliable items for assessment have not yet been fully developed around the world. It is not advisable at the moment to incorporate them in high stake examinations in a large scale. Rather they could help in the improvement of teaching if teachers could attempt to diagnose students' learning progress during their course of teaching. In fact, in the survey, students did express the hope that the teachers would frequently check whether they really understood the work done. For public examinations of General Mathematics at Certificate level, the creation of different papers to cater for different ability groups is worth exploring.

11.3.5. Students' interest in early years

The learning which takes place in the primary school years is of fundamental importance as it is there that students' interest in mathematics is still high. Students' motivation for learning should be cultivated so that gradually they can develop an interest in discovering mathematics for themselves. In this way, graduation from school can really become the starting point of life-long learning. Students' interest in learning can be cultivated by various means. The students' survey has indicated that one way is to show them how formulae come about and

how they are applied. Seeing the usefulness of mathematics in different professions and introducing various topics with examples close to students' experience could add relevance to the subject matter. However, it is essential to let student have first-hand experience of working on mathematics so that mathematical discovery would become an endeavour of their intellectual curiosity. It is, however, worthy to note that confining mathematics learning to artificially created real life situations could deprive the students of genuine mathematics learning.

11.3.6. Information technology

Information technology is not only something that our children should equip themselves with, but it can often provide a more effective way of learning. However, more research is needed on when and how information technology can effectively be used in mathematical instruction. Furthermore, one of the points revealed through this research is that teachers are in need of clear guidance in the use of information technology. Teachers need strong technical supports too, including hardware, software and time for preparation. Information technology can also have impact on the mathematics curriculum. Tedious computations are outdated and skills should be de-emphasised to leave room for conceptual understanding. We should reflect on what kind of mathematics should be learned. At present, many teachers have already recognised the motivational aspect of incorporating information technology into mathematics teaching, but few have recognised that the integration of information technology into mathematics learning may generate a completely new kind of mathematics experiences for students that will have strong, long-term influences on their ways of knowing and understanding mathematics.

11.3.7. The teachers

We need a strong teaching profession in which teachers can see the need and progress of their students and tailor-make learning experiences for them. Teachers' conceptions of mathematics and mathematics learning should be widened. To this end, the mathematics background in formal (pre-service) teacher education courses should be strengthened. We should have a long-term plan of institutionalised in-service programmes (rather than a piecemeal approach) so that teachers are given the encouragement, opportunity and support (for instance, provision of supply teachers) to upgrade themselves. We should create multiple channels so that teachers can develop, experiment and share various teaching strategies. This could be done in collaboration with professional bodies so as to

create a culture of collegiate exchange. All these can only be put into practice if the existing workload of the teacher is reduced.